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CALCULATING THE HIGH FREQUENCY RESISTANCE OF SINGLE AND DOUBLE LAYER TOROIDAL WINDINGS

by:

Bruce Carsten
Bruce Carsten Assoc., Inc.

for:

MICROMETALS, Inc.
Anaheim, CA

ABSTRACT

Finite Element Analysis (FEA) software was used to model the HF losses in single and double layer toroidal windings of solid round wire over a wide range of conditions. At high frequencies, where the wire diameter is greater than 1.1 to 1.4 “skin depths”, the single layer winding is found to have the lower loss. Formulas and graphs for calculating single layer loses were derived, including the effects of insulation thickness, increased winding pitch at the outside circumference, and proximity of the core.

INTRODUCTION

The low cost and minimal external field of toroidal inductors finds them many applications in RF equipment and switching power converters, ranging from resonant inductors to filter chokes. A drawback has been the difficulty of calculating winding losses at high frequencies, where the conductor diameter becomes comparable to or greater than a skin depth.

Reasonably accurate formulas are available for calculation of the HF resistance of solenoidal windings, notably derivations by Dowell [1] and Perry [2], with extensions to non-sinusoidal currents by Venkatraman [3], Carsten [4], and Vandelac & Ziogas [5]. These derivations are all based on simplifying assumptions which reduce a two or three dimensional problem to a one dimensional approximation of the physical winding, which is then amenable to analytical analysis. The simplifying assumptions are sufficiently valid that calculated resistances can be within ±5% to 10% of measured values in many practical cases.

The situation is somewhat more difficult with toroidal windings, where wire spacing changes radially on the toroid. The effective number of layers also changes radially when you use more than a “single” layer on the inside of the toroid. An extra complication is that toroidal windings are often placed very close to a magnetic core, which modifies the magnetic field pattern near the conductors, and thus the HF current distribution and winding resistance. The inherent multi-dimensional nature of toroidal windings renders analytical techniques inapplicable.

DERIVATION OF TOROIDAL WINDING RESISTANCE FORMULAS

Two principal approaches to determining the HF resistance of toroidal windings are empirical measurement and computer modeling, using finite element analysis programs (FEA). The measurement of HF resistance is feasible for specific instances, but investigating a sufficient number of cases to derive general formulas is only practical with a sufficiently powerful and efficient computer program.

Electromagnetic FEA software has improved markedly in the last ten years in terms of computer requirements, speed of operation and ease of use. The software used for this work was the ANSOFT Maxwell 2D Field Simulator, which proved invaluable. Running on a 133 MHz Pentium® computer (with a minimum of 16 Mbytes of RAM), analysis times ranged from typical values of 12 to 30 seconds up to 10 to 15 minutes in extreme cases. The latter occurred with multiple conductors and wire diameter to skin depth ratios of 100:1. Computation time was significantly increased by the reduction of the allowed field energy error by 100:1 (from 2% to 0.02%), which was found necessary to obtain high accuracy under these extreme conditions.

About 740 conditions were modeled to derive the winding resistance formulas presented here, with another several hundred used for exploratory purposes and to refine the approach.

CONDITIONS AND ASSUMPTIONS

The formulas derived assume “precision” toroidal windings of round wire, with complete and uniform coverage of the toroidal core. Conductors are assumed straight and unbowed, everywhere in contact with the core and/or each other. Only single and dual layer windings are considered, as illustrated in Figure 1. As will be shown later, single layer windings have a lower HF resistance than two (or more) winding layers of solid conductors. Thus the bulk of the formula derivation is devoted to accurate resistance calculations for single layer windings.

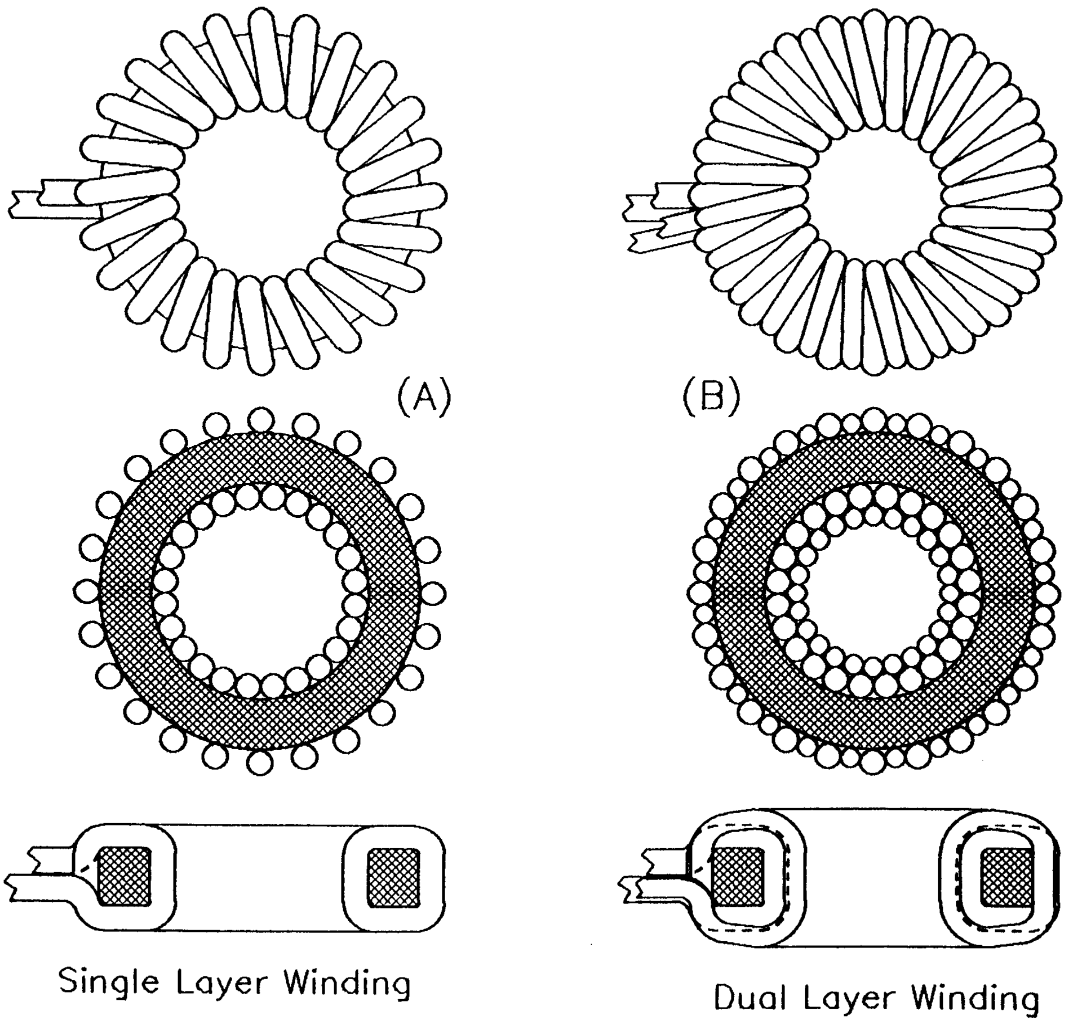


Fig. 1

"Precision" Toroidal Windings

Only a two dimensional FEA program was available, so a few simplifying assumptions still had to be made. The physical winding is essentially a two dimensional case on the inside and outside of a toroid of rectangular cross section, allowing conductor losses to be accurately modeled. Along the top and bottom of the winding the conductors are not parallel however, and create a three dimensional situation.

<u>The principal assumption</u> in this work is that the conductor leases vary approximately linearly with radius on the top and bottom of a toroidal winding of rectangular cross section, allowing the effective total winding AC to DC resistance ratio (Rac/Rdc) to be calculated as the algebraic mean (average) of the Rac/Rdc ratios derived for the inside and outside portions of the winding.

To the extent that this assumption is valid, the "form factor" of the rectangular cross section is not a consideration; the winding resistance formulas will apply to windings on a flat or "pancake" toroidal winding (Fig. 2a), a conventional toroid (2b), or on a highly cylindrical core (2c).

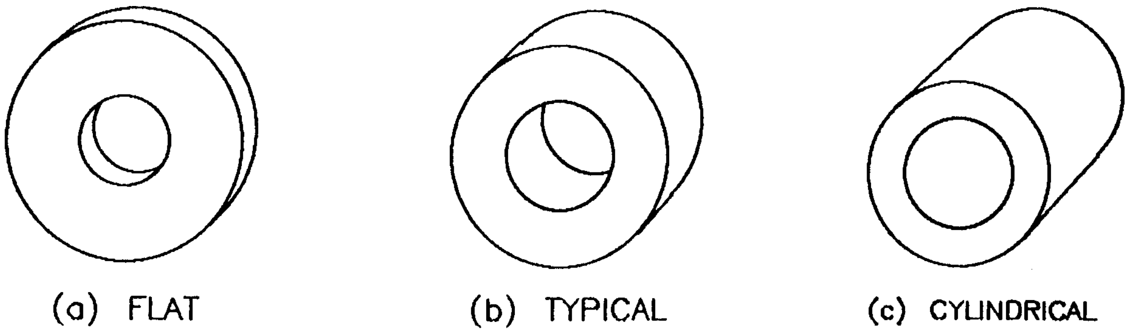


Figure 2

This assumption was checked by comparing the average inside and outside Rac/Rdc ratio with that for conditions at the mean radius. For precision single and double layer windings the assumption appeared to be reasonably accurate, and should contribute no more than a few percent error to a calculation.

Other assumptions are that magnet wire insulation thickness is 2% of the conductor diameter, that magnetic cores have no hysteresis or eddy current losses reflected back to the winding, and that winding "corner" effects can be neglected. Conductor size, resistivity and frequency effects are combined in the normalized, dimensionless Wire Diameter/Skin Depth ratio.

SINGLE LAYER TOROIDAL WINDING RESISTANCE

Losses in windings with an "air" or non-magnetic core are first considered. Given the previous assumptions, the problem becomes one of determining the HF resistance of a single layer of round wires with variable spacing, assuming the "solenoidal" condition of all magnetic flux on one side of the plane of the conductors.

For round conductors in near contact (zero insulation thickness), it was found that the Rac/Rdc ratio vs. frequency was very similar to that of a foil of the same conductivity and a thickness near 84.4% of the wire diameter. The precise equivalent foil thickness for the same Rac/Rdc vs. wire diameter/skin depth is shown in Figure 3, varying from 82.7% to 86.1% of the wire diameter, within ±2% of 84.4%.

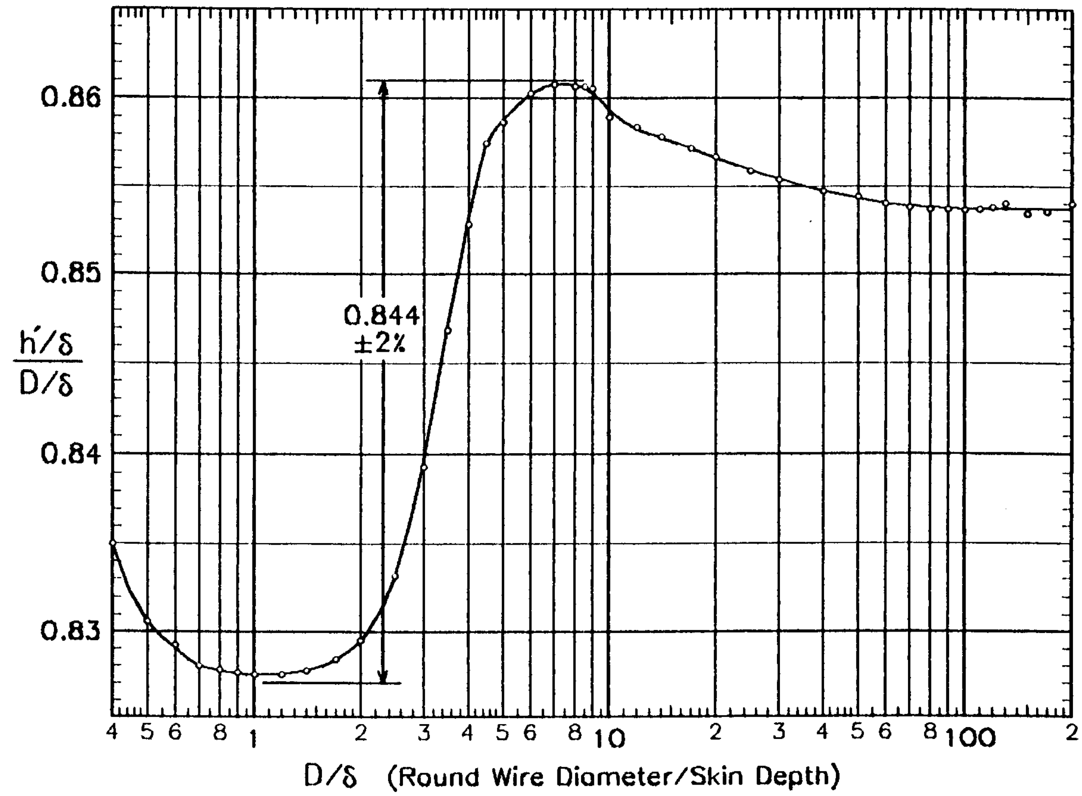


Fig. 3

"Equivalent Foil" Thickness (h′/δ) to Round Wire Diameter (D/δ) Ratio, vs Wire D/δ.

When wire spacing effects on HF resistance were evaluated, it was found convenient to separate the AC resistance (Rac) into DC resistance (Rdc) and eddy current resistance (Rec) components, such that:

$$Rac = Rdc + Rec \qquad (1)$$

which can be rewritten as:

$$Rec = Rac - Rdc \qquad (2)$$

or:

$$Rec/Rdc = Rac/Rdc - 1 \qquad (3)$$

It was found that the "eddy current" component of the resistance dropped significantly as the wire spacing increased, and was about inversely proportional to the wire pitch/diameter ratio "P/D". This was particularly true at higher frequencies, and for P/D = 2 (which is usually the case). At lower frequencies the eddy current resistance is a smaller portion of the total resistance, so the error of this approximation in calculating Rac is typically less than ±10%. For more accurate calculations a spacing correction factor K1 from Fig. 4 is used.

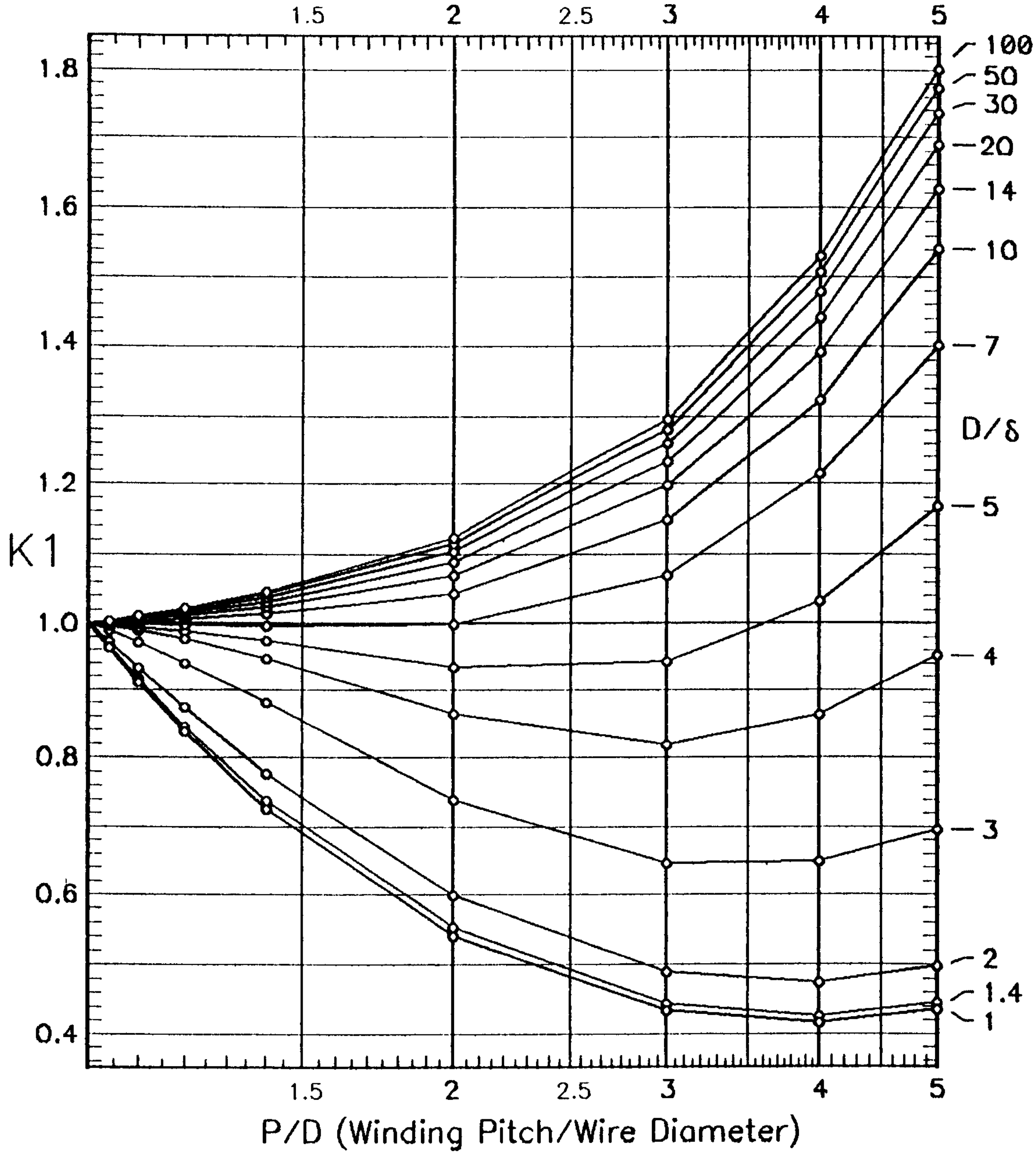


Fig. 4

Single Layer Wire Spacing Correction Factor K1

(Correction to the Assumption that Rec ∝ D/P)

With an "air" core, the HF winding current tends to concentrate on the "inside" portion of the wire circumference (Figure 5a). The presence of a magnetic core near the winding causes the current to spread up the "sides" of the conductor (Fig. 5b), reducing the HF resistance.

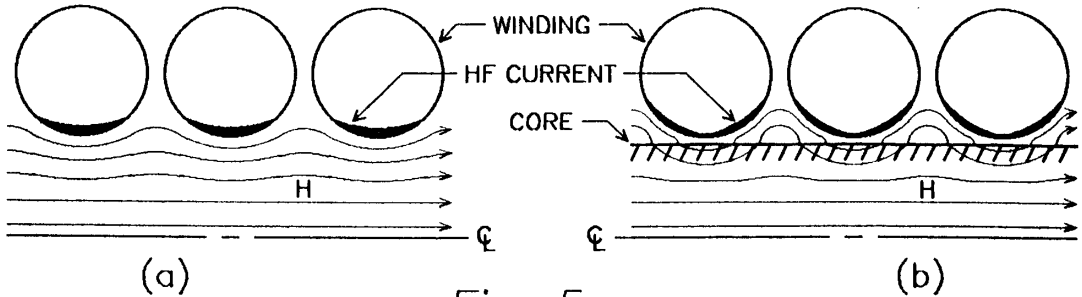


Fig. 5

HIGH FREQUENCY CURRENT DISTRIBUTIONS

Significant reductions in eddy current resistance can occur under conditions of large winding P/D ratio and a nearby high permeability core, but only for D/δ ‹ 7, where the contribution of eddy current losses is modest. The effect on total resistance Rac is less than a 13% reduction in loss for widely spaced wires at high frequencies, as shown in Figure 8. The beneficial effect decreases with reduced wire spacing, increased core to winding distance, and/or very low permeability cores (u < 10).

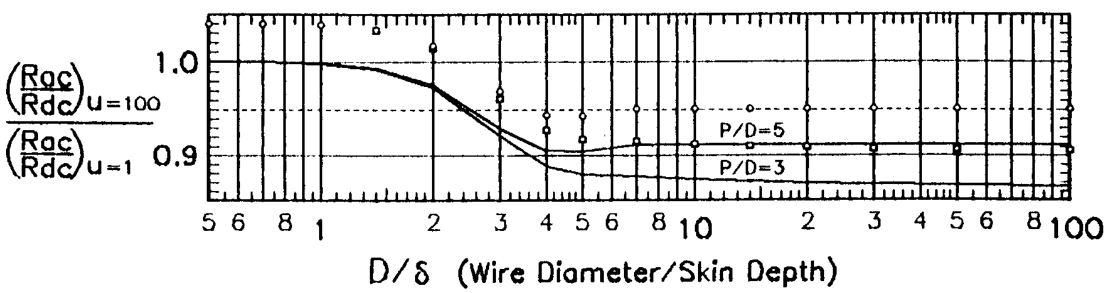


Fig. 6

Actual Core Proximity Effects on Rac for Core u = 100

The core proximity effect is thus never great, but is often large enough to require consideration for the highest obtainable accuracy. Unfortunately, the core proximity effect is a complex function of four variables: frequency (or wire diameter/skin depth), wire P/D ratio, core permeability, and core to winding spacing. These effects are combined in a second correction factor "K2", which is found in the graphs of Figures 7, 8 and 9.

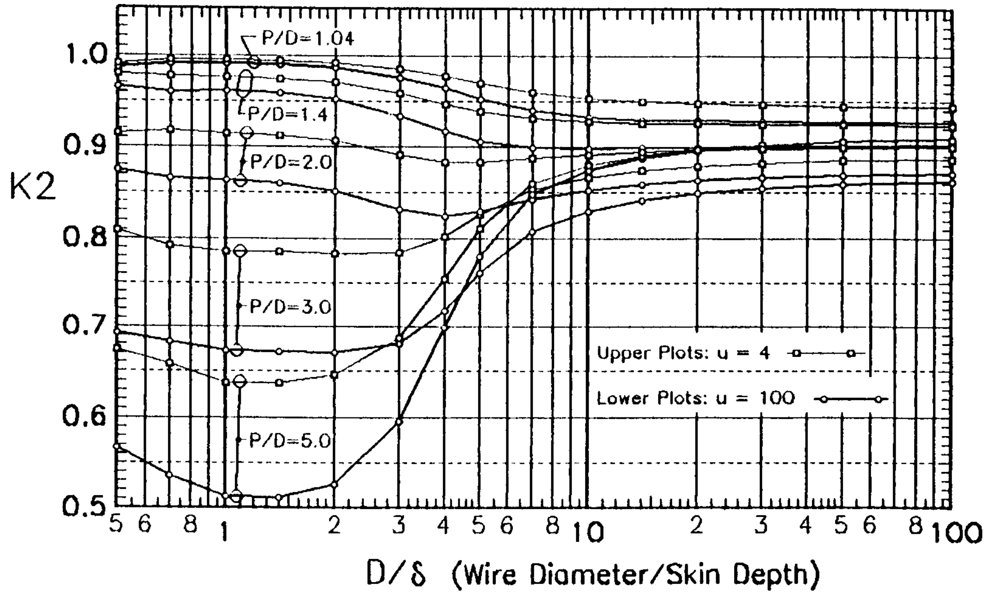


Fig. 7 Core Proximity Correction Factor K2

For Single Layer Windings Close to Core (s/D = 0.04)

<u>Single Layer Winding Resistance Calculation Procedure</u>

The net winding Rac/Rdc ratio is calculated as the average of Rac/Rdc for the winding geometries at the inner and outer diameters of the toroid, using the formulas below and the figures noted. The following definitions are used:

Rac	=	AC Winding Resistance
Rdc	=	DC Winding Resistance
Rec	=	Eddy Current Winding Resistance = Rac - Rdc
f	=	Frequency
D	=	Conductor Diameter (without Insulation)
δ	=	Skin Depth at Frequency “f”
P	=	Winding Pitch (Conductor Center-Center Distance)
s	=	Wire to Core Spacing
u	=	Core Permeability
h'	=	“Equivalent Foil” Thickness
K1	=	Winding P/D Correction Factor
K2	=	Core Proximity Correction Factor

1) For the conductor diameter and frequency of interest, calculate the wire diameter/skin depth ratio D/ δ, where skin depth (in cm) for copper wire can be calculated from:

$\delta = 6.6/f^{1/2}$ at 20°C (ρ = 1.70 x 10^{-6} Ω cm) . . . . (4)

$\delta = 7.1/ f^{1/2}$ at 60°C (ρ = 1.99 x 10^{-6} Ω cm) . . . . (5)

$\delta = 7.6/ f^{1/2}$ at 100°C (ρ = 2.28 x 10^{-6} Ω cm) . . . . (6)

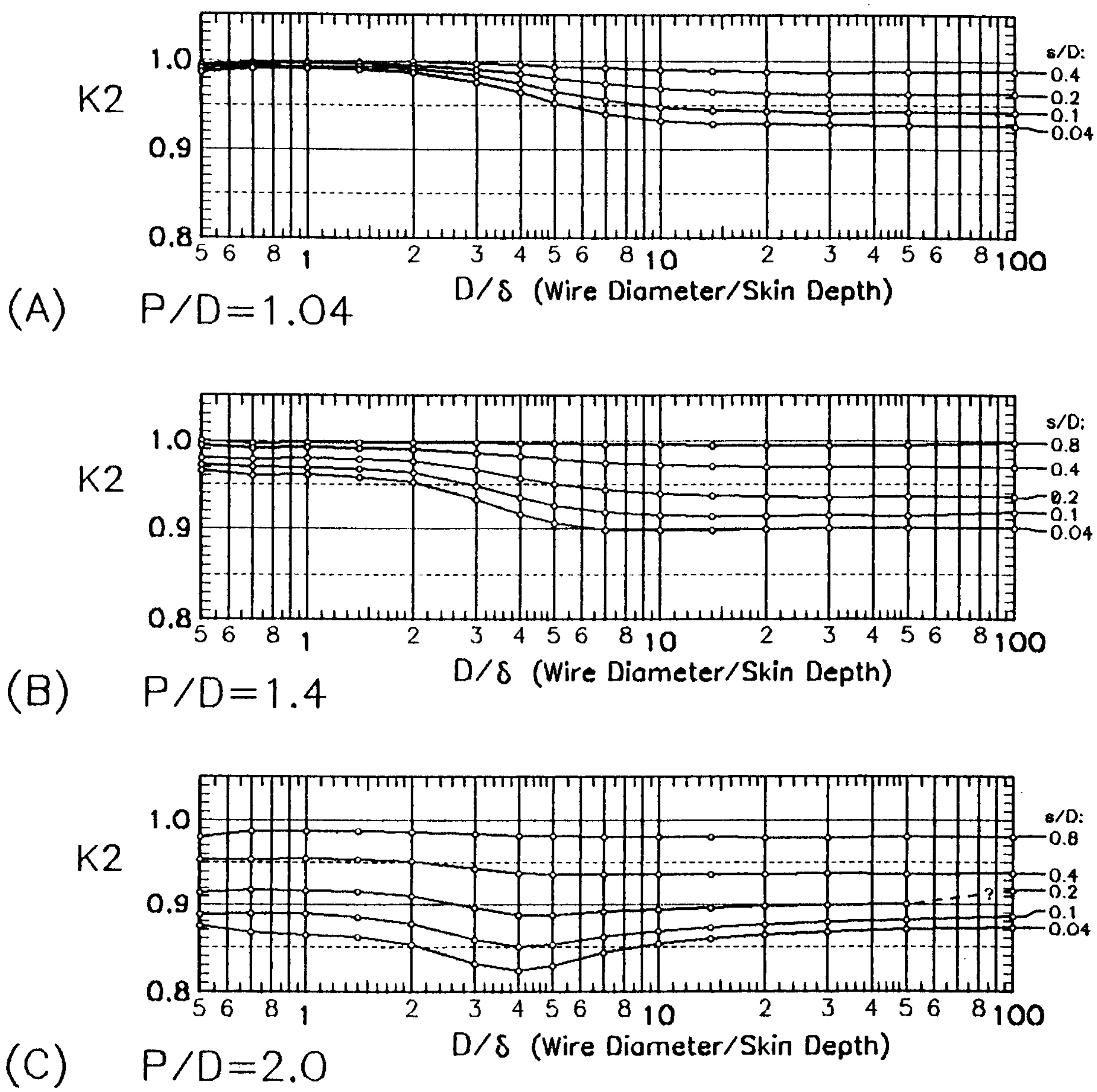


Fig. 8

Core Proximity Correction Factor K2, Core u=100

2) The eddy current resistance ratio Rec/Rdc is calculated for an "equivalent foil" of thickness h' (for P/D = 1) with equation (8) adapted from Dowell [1]. It can either be assumed that h' = 0.884(D), or a more precise value obtained from Fig. 3.

For $X = h'/\delta$ (7)

$$\frac{Rec}{Rdc} = X\frac{e^{2X} - e^{-2X} + 2\sin(2X)}{e^{2X} + e^{-2X} - 2\cos(2X)} - 1 \qquad (8)$$

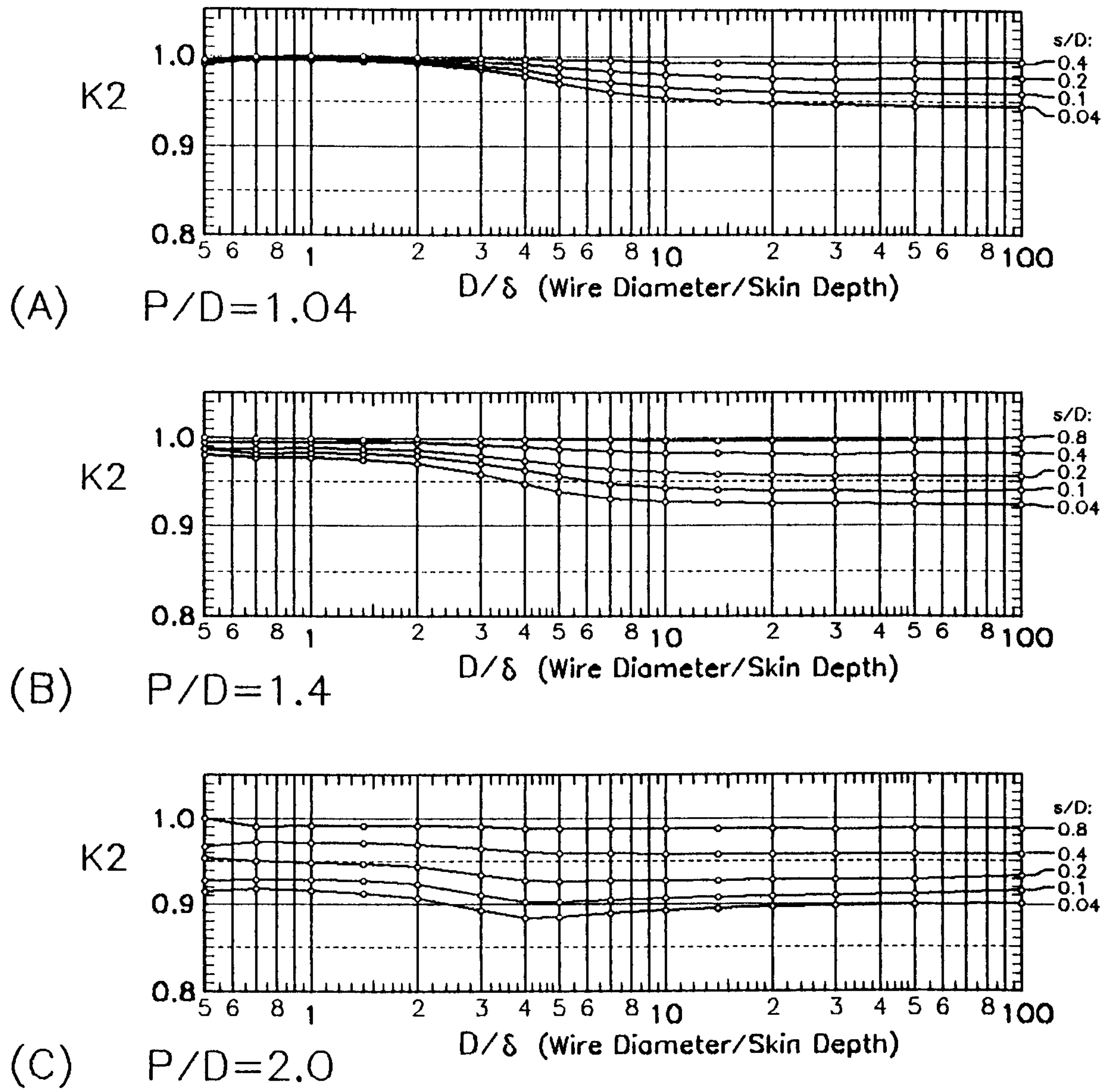


Fig. 9

Core Proximity Correction Factor K2, Core u=4

3) Determine the Winding Spacing Correction Factor K1 from the curves in Figure 4. K1 must be obtained for the P/D ratios on both the inside and outside diameters of the toroidal winding.

4) Determine the Core Proximity Correction Factors K2 from Figure 7, 8 or 9 for both the inside and outside diameters of the toroid.

Strictly speaking, Fig. 7 applies only to windings wound directly on top of a magnetic core; ie, with a winding to core spacing of 4% of the wire diameter. As this is just twice the typical thickness of magnet wire insulation, this spacing will only be approached with large wires, due to the usual insulating coating on the core. As noted earlier, the relatively small

K2 values at large P/D and small D/ δ are deceptive; due to the small contribution of Rec to Rac in this region, the actual Rac/Rdc is much less affected, as shown previously in Fig. 6.

Figure 8 provides K2 values for core to winding spacings (s/D) up to 0.8, a winding P/D up to 2.0, and core permeability of 100. Corresponding K2 values for a core permeability of 4 are given in Figure 9. Permeabilities between 1 and 4 are uncommon, and were not modeled. Core permeabilities in the range of 5 to 10 have intermediate core proximity effects, while permeabilities much above 10 are "high" compared to free space, and effects will be similar to the curves for u = 100.

5) Calculate Rac/Rdc for the winding portions on the inside and outside diameters of the toroid, using the above information in the formula:

$$\frac{Rac}{Rdc} = 1 + \left(\frac{Rec}{Rdc}\right)\left(\frac{D}{P}\right)(K1)(K2) \qquad (9)$$

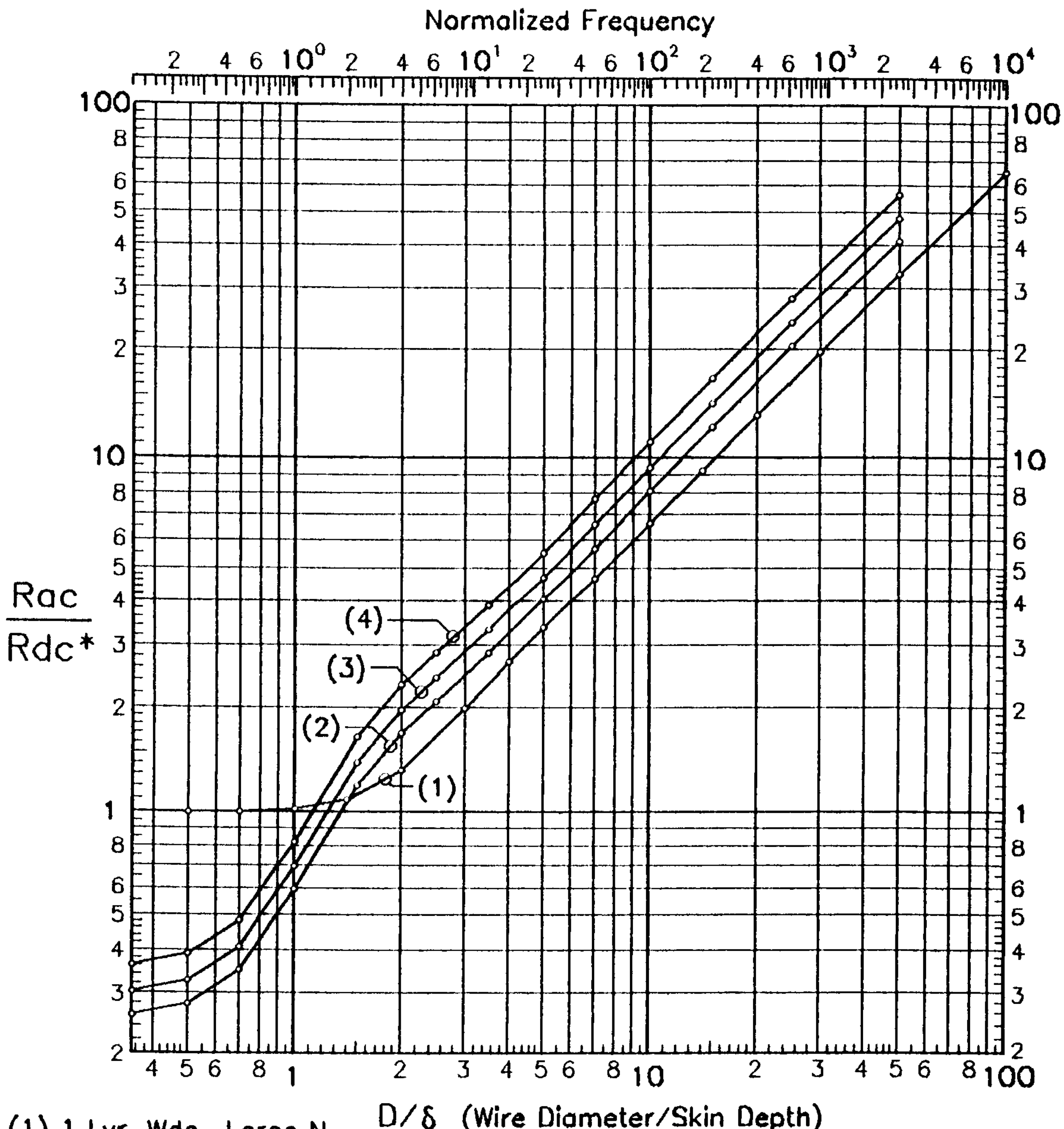
6) Calculate the effective Rac/Rdc for the full winding as the average of the Rac/Rdc ratios for the inside diameter (id) and outside diameter (od) portions of the winding, and multiply by the DC resistance of the winding to obtain the AC resistance at frequency "f".

$$Rac = \frac{1}{2}\left(\frac{Rac_{(id)}}{Rdc} + \frac{Rac_{(od)}}{Rdc}\right)Rdc \qquad (10)$$

DOUBLE LAYER TOROIDAL WINDINGS

Two layer toroidal windings "put a lot more copper" on the core, and thus would appear to have a much lower resistance. This is particularly evident with the two layers in series, as the wire diameter can be nearly twice as large as with a full single layer for the same number of turns, and thus the DC resistance is nearly 1/4 as great.

But, as the saying goes, looks can be deceiving. At higher frequencies the current only flows one skin depth deep on a portion of the wire surface, "wasting" most of the copper. The wider conductors do lower the conduction resistance, but the second layer currents induce eddy current losses in the first layer which more than make up for the lower conduction losses. These effects are shown in Figure 10.



(1) 1 Lyr. Wdg., Large N

(2) 2 Lyr. Wdg., Large N

(3) 2 Lyr. Wdg.; Σ N = 94, Δ AWG = 1

(3) 2 Lyr. Wdg.; Σ N = 48, Δ AWG = 2

Fig. 10

Ratio of AC Winding Resistance "Rac" (for 1 and 2 Layer Windings) to the DC Resistance "Rdc*" (for a Single Layer Winding) vs D/δ (in the Single Layer Winding), Assuming Equal Turns and Winding Layers in Series.

Curve (1) in Figure 10 is the AC/DC resistance ratio vs. D/ δ for a single layer toroidal winding, assuming;

- Many turns of relatively fine wire;
- A core OD/ID ratio of nearly 2.0;
- An “air” core (no core proximity effects).

Curves (2), (3) and (4) are for various two layer windings. In each case Rac/Rdc* is the ratio of the AC resistance of the two layer winding to the DC resistance of a single layer winding with the same number of turns on the same “core” measured at the same frequency for which D/δ is calculated for the single layer winding.

Curve (2) is also for a winding with a large number of turns, allowing the second layer of wire to be essentially the same size as the first layer, have the same “length of mean turn” (lmt), and allowing the turns of the “second” layer to lie between those of the first on the outside diameter.

Curve (3) is for a 94 turn winding. With 47 turns on each layer, the second layer must be just one AWG size smaller than the first layer, and the lmt is typically about 7% greater.

Curve (4) is for a 48 turn winding with 24 turns on each layer. The second layer must now be two AWG sizes smaller than the first layer, and the lmt is about 14% greater.

The winding of curve (2) is a “best case” situation, where the DC resistance of the two layer winding is 25% of that for a single layer winding. When the frequency is high enough for the wire of the single layer winding to be 1.4 skin depths in diameter, however, the AC resistance of the dual layer winding becomes greater. Most practical toroidal windings will fall somewhere between curves (2) and (4), with a loss crossover at a single layer D/δ of 1.1 to 1.4.

Double layers with the windings in parallel will not show quite the same increase in resistance over a single layer winding, as at high frequencies the current will all tend to flow on the inner winding. (In a nutshell, HF AC current on the inner winding induces the same “back EMF” on the outer winding, as all of the flux produced by the inner winding current is encompassed by the outer winding. There is then zero voltage across the finite leakage inductance between the two windings, and no current flows in the outer winding.)

There is some eddy current loss induced in the second layer at the outside diameter, however, so the total loss is still slightly greater than for a single layer. The AC resistance with paralleled windings is difficult to impossible to model with 2D software, as the current distribution at intermediate frequencies is indeterminate.

Precision double layer toroidal windings are also very difficult to make; the second layer winding tries to “wedge” the turns of the first layer apart on the corners of the outside diameter, leading to a progressive distortion of the winding unless the evenly spaced turns of the first layer are held in place with an adhesive, or some type of form or jig.

For those interested in attempting precision dual layer toroidal windings, a few useful relationships derived for this research are presented in Appendix A.

CONCLUSIONS

A Finite Element Analysis software program has been used to derive formulas and graphs for the calculation of high frequency toroidal winding losses. These formulas for sine waves can be applied to the triangular wave current component typically found in filter inductors with little error, but more “harmonic rich” waveforms will require application of the techniques described in [3], [4] and [6]. Also, note that losses from DC and RMS AC currents are calculated independently.

It was found that a uniform single layer winding provided the minimum lose with solid conductors at high frequencies. It was also found that spacing round wires significantly reduced the eddy current loses, but not sufficiently to reduce the total loss per winding width.

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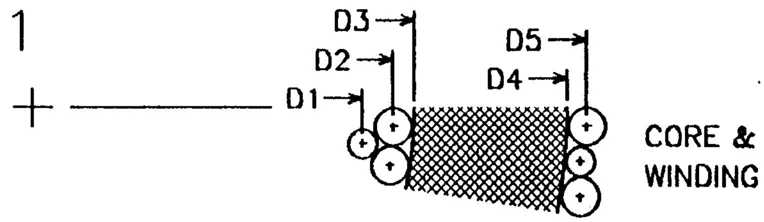
Appendix A

RELATIONSHIPS AND FORMULAS
for
PRECISION DUAL LAYER TOROIDAL WINDINGS

Fig. 11



Let: n = Number of Turns/Layer
w = 1st Layer Wire Dia. (Over Insulation)
x = Numerical Change of AWG Wire Size of 2nd Layer
(Wire Diameter changes 12.3% /AWG Guage)

$$D2 = \frac{w}{\mathrm{Sin}\left(\frac{180}{n}\right)} \qquad (11)$$

$$D3 = w\left(\frac{1}{\mathrm{Sin}\left(\frac{180}{n}\right)} + 1\right) = D2 + w \qquad (12)$$

$$D5 \geq D2\left(\frac{1}{1.123^{x}} + 1\right) = \frac{w}{\mathrm{Sin}\left(\frac{180}{n}\right)}\left(\frac{1}{1.123^{x}} + 1\right) \qquad (13)$$

$$D4 = D5 - w \qquad (14)$$

$$D1 = w\left(\frac{1}{\mathrm{Tan}\left(\frac{180}{n}\right)} - \frac{\sqrt{2\,(1.123)^{x}+1}}{1.123^{x}}\right) \qquad (15)$$

$$\frac{D4}{D3} \geq \frac{\frac{1}{1.123^{x}} + 1 - \mathrm{Sin}\left(\frac{180}{n}\right)}{1 + \mathrm{Sin}\left(\frac{180}{n}\right)} \qquad (16)$$

$$\text{Wire Diameter (inches)} = 0.3249 \times 10^{-(AWG/19.86)} \qquad (17)$$